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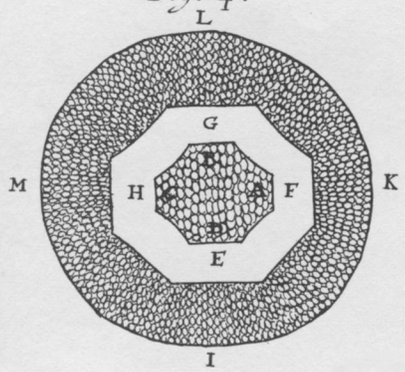
Fig: 4.



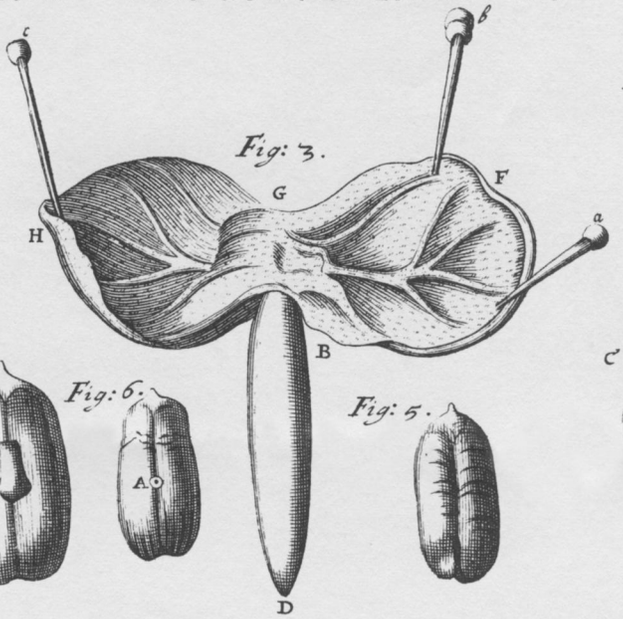
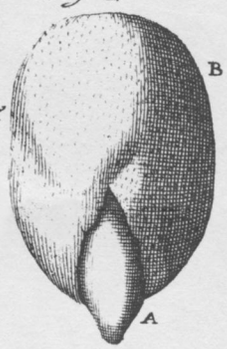


Fig: 1.

Fig: 2.



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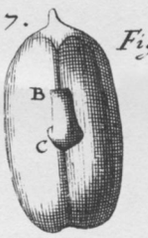

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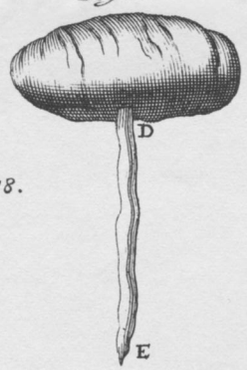

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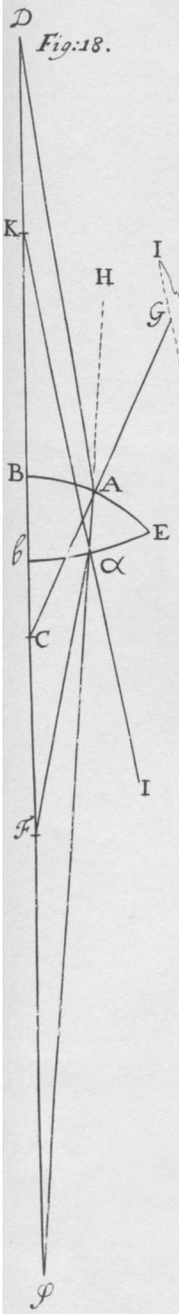

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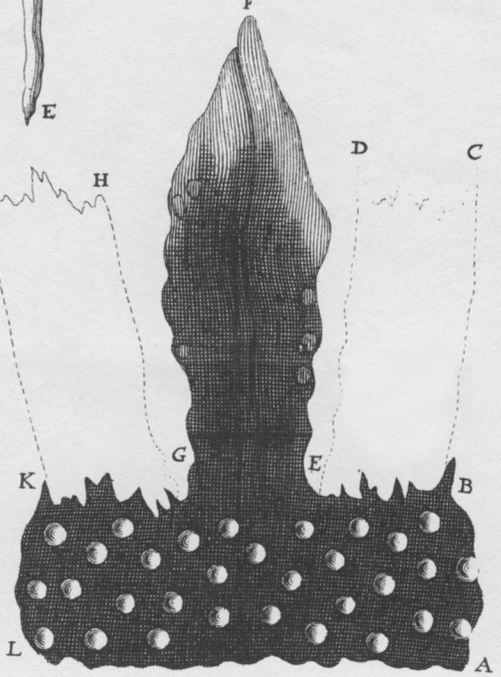

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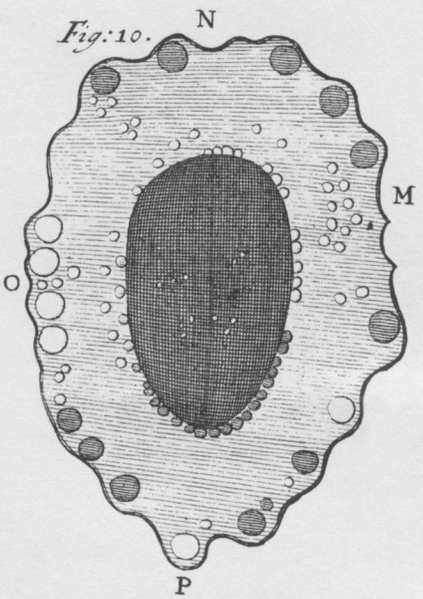


Fig:B.

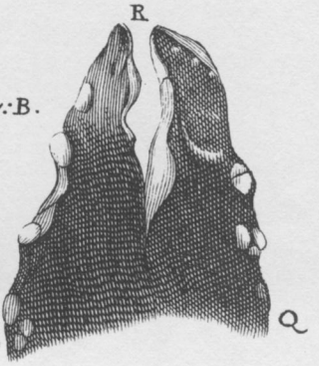
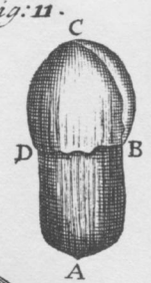


Fig:12.

Fig:11.



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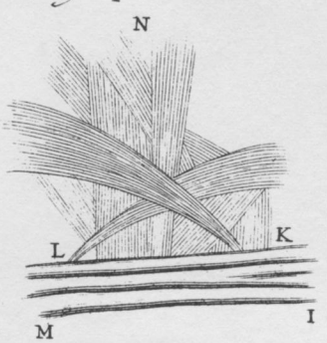

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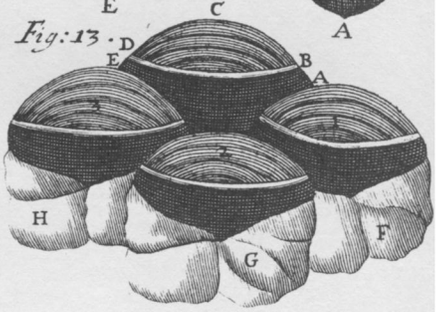

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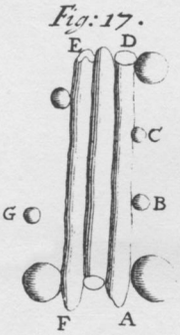

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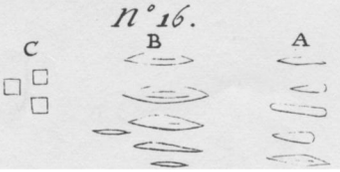

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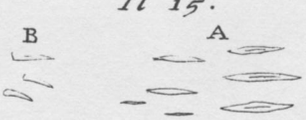

bule of Air as *B* has been above a larger as *A*, which Globule has not riſen upward to *C*, and ſo to *D*, but been thruſt downwards to *A*, whence it was diſtant about two hair's breadths, and immediately upon touching united therewith. I have likewiſe obſerved, that a little Air-bubble as *G*, looſening itſelf from the ſtraw, when a larger Bubble, as *F*. was underneath it, has there reſted immovable in the Liquor, when at the ſame time other much ſmaller Bubbles have riſen to the top thereof. The reaſon of the ſtanding ſtill of the Bubble *G*, I ſuppoſe was from a double motion it is impelled to, the one upwards from its being ſpecifically higher than the Liquor, the other downwards, by which it was protruded, to joyn with the other larger Bubble *F*. Tho' I have ſeen ſeveral Effects of Sympathy, if we may ſo call it, yet I never ſaw any ſo manifeſt as this, of the deſcending of a Bubble contrary to its levity, to unite with another.

IV. *An Inſtance of the Excellence of the Modern* ALGEBRA, *in the Reſolution of the Problem of finding the* Foci *of Optick Glaſſes univerſally. By* E. Halley, *S. R. S.*.

THE Excellence of the *Modern Geometry* is in nothing more evident, than in thoſe full and adequate Solutions it gives to Problems; repreſenting all the poſſible Caſes at one view, and in one general Theorem many times comprehending whole Sciences; which deduced at length into Propoſitions, and demonſtrated after the manner of the *Ancients*, might well become the Subjects of large Treatiſes: For whatſoever Theorem ſolves the moſt complicated Problem of the kind, does with a due Reduction reach all the ſubordinate Caſes. Of this

I now deſign to give a notable Inſtance in the Doctrine of *Dioptricks*.

This Dioptrick Problem is that of finding the *Focus* of any ſort of *Lens*, expoſed either to Converging, Diverging or parallel Rays of Light, proceeding from, or tending to a given Point in the *Axis* of the *Lens*, be the *ratio* of *Refraction* what it will, according to the nature of the Tranſparent Material whereof the *Lens* is formed, and alſo with allowance for the thickneſs of the *Lens* between the *Vertices* of the two Spherical Segments. This Problem being ſolved in one Caſe, *mutatis mutandis* will exhibit Theorems for all the poſſable Caſes, whether the *Lens* be *Double-Convex* or *Double-Concave*, *Plano-Convex* or *Plano-Concave*, or *Convexo-Concave*, which ſort are uſually called *Meniſci*. But this is only to be underſtood of thoſe Beams which are neareſt to the *Axis* of the *Lens*, ſo as to occaſion no ſenſible difference by their Inclination thereto; and the *Focus* here formed is by *Dioptrick Writers* commonly called the principal *Focus*, being that of uſe in *Teleſcopes* and *Microſcopes*.

Let then (in Fig. 18.) BEβ be a double Convex *Lens*, C the Center of the Segment EB, and K the Center of the Segment Eβ, Bβ the thickneſs of the *Lens*, D a point in the *Axis* of the *Lens*; and it is required to find the point F, at which the Beams proceeding from the point D, are collected therein, the *ratio* of Refraction being as m to n. Let the diſtance of the object DB = DA = d (the point A being ſuppoſed the ſame with B, but taken at a diſtance therefrom, to prevent the coincidence of ſo many Lines) the *Radius* of the Segment towards the Object CB or CA = r, and the *Radius* of the Segment from the Object Kβ or Kα = ρ, and let Bβ the thickneſs of the *Lens* be = t, and then let the Sine of the Angle of incidence DAG be to the ſine of the refracted Angle HAG or CAφ as m to n; And in very ſmall Angles the Angles them-

themſelves will be in the ſame proportion; whence it will follow that,

As d to r, ſo the Angle at C to the Angle at D, and $d+r$ will be as the Angle of incidence GAD; and again as m to n, ſo $d+r$ to $\frac{dn+rn}{m}$ which will be as the Angle GAH = CAφ. This being taken from ACD which is as d, will leave $\frac{m-nd-nr}{m}$ analogous to the Angle AφD, and the ſides being in this caſe proportional to the Angles they ſubtend, it will follow, that as the Angle AφD is to the Angle ADAφ, ſo is the ſide AD or BD to Aφ or Bφ: that is Bφ will be $=\frac{mdr}{m-nd-nr}$ which ſhews in what point the beams proceeding from D would be collected by means of the firſt Refraction; but if nr cannot be ſubſtracted from $m-nd$, it follows that the Beams after Refraction do ſtill paſs on diverging, and the point φ is on the ſame ſide of the *Lens* beyond D. But if nr be equal to $m-nd$ then they proceed parallel to the *Axis*, and the point φ is infinitely diſtant.

The point φ being found as before, and Bφ—Bβ being given, which we will call δ, it follows by a proceſs like the former, that βF or the focal diſtance ſought, is equal to $\frac{\delta\rho n}{m-n\delta+m\rho}=f$. And in the room of δ ſubſtituting Bφ—Bβ $=\frac{mdr}{m-nd-nr}-t$, putting p for $\frac{n}{m-n}$ after due reduction this following Equation will ariſe,

$$\frac{mpdr\rho-nd\rho t+npr\rho t}{mdr+md\rho-mpr\rho-m-ndt+nrt}=f.$$

Which Theorem, however it may ſeem operoſe,

is

is not ſo, conſidering the great number of *data* that enter the Queſtion, and that one half of the terms ariſe from our taking in the thickneſs of the *Lens*, which in moſt caſes can produce no great effect, however it was neceſſary to conſider it, to make our Rule perfect. If therefore the *Lens* conſiſt of *Glaſs*, whoſe Refraction is as 3 to 2 'twill be $\frac{6\,dr\rho - 2\,d\rho t + 4\,r\rho t}{3\,dr + 3\,d\rho - 6\,r\rho - dt + 2\,rt} = f.$ If of *Water*, whoſe Refraction is as 4 to 3 the Theorem will ſtand thus $\frac{12\,dr\rho - 3\,d\rho t + 9\,r\rho t.}{4\,dr + 4\,d\rho - 12\,r\rho - dt + 3\,rt} = f.$ If it could be made of *Diamant*, whoſe Refraction is as 5 to 2, it would be $\frac{\frac{10}{3}\,dr\rho - 2\,d\rho t + \frac{4}{3}\,r\rho t}{5\,dr + 5\,d\rho - \frac{10}{3}\,r\rho - 3\,dt + 2\,rt} = f.$ And this is the univerſal Rule for the *foci* of double Convex Glaſſes expoſed to Diverging Rays. But if the thickneſs of the *Lens* be rejected as not ſenſible, the Rule will be much ſhorter, *viz.* $\frac{p\,dr\rho}{dr + d\rho - \rho r\rho} = f$, or in Glaſs $\frac{2\,dr\rho}{dr + d\rho - 2r\rho} = f.$ all the terms wherein t is found being omitted, as equal to nothing. In this caſe, if d be ſo ſmall, as that $2r\rho$ exceed $dr + d\rho$, then will it be $-f$, or the *focus* will be Negative, which ſhews that the Beams after both Refractions ſtill proceed Diverging.

To bring this to the other Caſes, as of Converging Beams, or of Concave Glaſſes, the Rule is ever compoſed of the ſame terms, only changing the ſigns of + and −; for the diſtance of the point of Concourſe of converging Beams, from the point B, or the firſt ſurface of the *Lens*, I call a negative diſtance or $-d$; and the Radius of a Concave *Lens* I call a negative Radius or $-r$ if it be the firſt ſurface, and $-\rho$, if it be the ſecond ſurface. Let then converging Beams fall on a double Con-

vex

vex of Glaſs, and the Theorem will ſtand thus $\frac{-2drp}{-dr-dp-2rp} = +f$. which ſhews that in this caſe the *Focus* is always affirmative.

If the *Lens* were a *Meniſcus* of Glaſs, expoſed to diverging Beams, the Rule is $\frac{-2drp}{-dr+dp+2rp} = f$. which is affirmative when $2rp$ is leſs than $dr-dp$, otherwiſe negative: But in the caſe of converging Beams falling on the ſame *Meniſcus*, 'twill be $\frac{+2drp}{+dr-dp+2rp} = f$. and it will be $+f$ whilſt $dp-dr$ is leſs than $2rp$, but if it be greater than $2rp$, it will always be found negative or $-f$. If the *Lens* be double Concave, the *focus* of converging Beams is negative, where it was affirmative in the caſe of diverging Beams on a double Convex, *viz.* $\frac{-2drp}{+dr+dp-2rp} = f$, which is affirmative only when $2rp$ exceeds $dr+dp$: But diverging Beams paſſing a double Concave have always a negative *focus*, *viz.* $\frac{-2drp}{+dr+dp+2rp} = -f$.

The Theorems for Converging Beams are principally of uſe to determine the *focus* reſulting from any ſort of *Lens* placed in a Teleſcope, between the *focus* of the Object-glaſs and the Glaſs it ſelf; the diſtance between the ſaid *focus* of the Object-glaſs and the interpoſed *Lens* being made $=-d$.

I here ſuppoſe my Reader acquainted with the Rules of Analytical Multiplication and Diviſion, as that $+$ multiplied by $+$ makes the product $+$, $+$ by $-$ makes $-$, and $-$ by $-$ makes $+$, ſo dividing $+$ by $+$ makes the Quote $+$, $+$ by $-$ makes $-$, and $-$ by $-$ makes $+$, which will be neceſſary to be underſtood in the preceding Examples.

In

In caſe the Beams are parallel, as coming from an infinite diſtance, (which is ſuppoſed in the caſe of Teleſcopes) then will d be ſuppoſed infinite, and in the Theorem $\frac{p\,d\,\rho\,r}{d\,r \pm d\,\rho - p\,r\,\rho}$ the Term $p\,r\,\rho$ vaniſhes, as being finite, which is no part of the other infinite terms and dividing the remainder by the infinite part d, the Theorem will ſtand thus $\frac{p\,\rho\,r}{r \pm \rho} = f$, or in Glaſs, $\frac{2\,r\,\rho}{r \pm \varrho} = f$.

In caſe the *Lens* were *Plano-Convex* expoſed to diverging Beams, inſtead of $\frac{p\,d\,\rho\,r}{d\,r \pm d\,\varrho - p\,r\,\varrho}$, r being infinite, it will be $\frac{p\,d\,\varrho}{d - p\,\varrho} = f$. or $\frac{2\,d\,\varrho}{d - 2\,\varrho} = f$, if the *Lens* be Glaſs.

If the *Lens* be Double-Convex, and r be equal to ϱ, as being formed of Segments of equal Spheres, then will $\frac{p\,d\,\varrho\,r}{d\,r \pm d\,\varrho - p\,r\,\varrho}$ be reduced to $\frac{p\,d\,r}{2\,d - p\,r} = f$; and in caſe d be infinite, then it will yet be farther contracted to $\frac{1}{2}\,p\,r$, and p being $= \frac{n}{m - n}$ the focal diſtance in Glaſs will be $= r$, in Water $1\frac{1}{2}\,r$, but in Diamant $\frac{1}{2}\,r$.

I am ſenſible that theſe Examples are too much for the compleat Analyſt, though I fear too little for the leſs Skilful, it being very hard, if poſſible, in ſuch matters, ſo to write as to give ſatisfaction to both; or to pleaſe the one, and inſtruct the other. But this may ſuffice to ſhew the extent of our Theorem, and how eaſy a Reduction adapts any one caſe to all the reſt.

Nor is this only uſeful to diſcover the *focus* from the other propoſed *data*, but from the *focus* given, we may thereby determine the diſtance of the Object, or from the *focus* and diſtance given, we may find of what Sphere it is requiſite to take another Segment, to make any given Segment of another Sphere caſt the beams from the diſtance d to the *focus* f. As likewiſe from the *Lens*, *focus*, and diſtance given, to find the *ratio* of Refraction, or of m to n, requiſite to anſwer thoſe *data*. All which it is obvious, are fully determined from the Equation we have hitherto uſed, *viz.* $p\,d\varrho\,r = drf + d\varrho f - pr\varrho f$, for to find d the Theorem is

$$\frac{pr\varrho f}{rf + \varrho f - p\varrho r} = d,$$ the diſtance of the Object.

For ϱ the Rule is $\frac{drf}{p\,dr + df + prf} = \varrho$

But for p will be $\frac{drf + d\varrho f}{d\varrho r + f\varrho r} = p$, which latter determines the *ratio* of Refraction, m being to n as $1 + p$ to p.

I ſhall not expatiate on theſe Particulars, but leave them for the exerciſe of thoſe that are deſirous to be informed in Optical Matters, which I am bold to ſay are comprehended in theſe three Rule, as fully as the moſt inquiſitive can deſire them, and in all poſſible caſes; regard being had to the Signs + and −, as in the former caſes of finding the *focus*. I ſhall only ſhew two conſiderable uſes of them; the one to find the diſtance whereat an Object, being placed ſhall by a given *Lens* be repreſented in a *Species* as large as the Object it ſelf, which may be of ſingular uſe, in drawing Faces, and other things in their true Magnitude, by tranſmitting the *Species* by a Glaſs into a dark Room, which will not only give the true Figure and Shades, but even the Colours themſelves, almoſt as vivid as the Life. In this caſe

caſe d is equal to f, and ſubſtituting d for f in the Equation, we ſhall have $p\,d\,r\,\varrho = d\,d\,r \pm d\,d\varrho - d\,p\,\varrho\,r$, and dividing all by d. $p\,r\,\varrho = d\,r + d\varrho - p\,r\,\rho$, that is $\frac{2\,p\,r\,\varrho}{r \pm \varrho} = d$; but if the two Convexities be of the ſame Sphere ſo as $r = \varrho$ then will the diſtance be $= p\,r$, that is, if the *Lens* be Glaſs $= 2\,r$, ſo that if an Object be placed at the Diameter of the Sphere diſtant, in this caſe the *focus* will be as far within as the Object is without, and the *Species* repreſented thereby will be as big as the Life; but if it were a *Plano-Convex*, the ſame diſtance will be $= 2\,p\,r$, or in Glaſs to four times the *Radius* of the Convexity; but of this method I may perhaps entertain the Curious in ſome other Tranſaction, and ſhew how to magnifie or diminiſh an Object in any proportion aſſigned, (which yet will be obvious enough from what is here delivered) as likewiſe how to erect the Object which in this method is repreſented inverted.

A ſecond uſe is to find what Convexity or Concavity is required, to make a vaſtly diſtant Object be repreſented at a given *focus*, after the one ſurface of the *Lens* is formed; which is but a Corollary of our Theorem for finding ϱ, having p, d, r and f given; for d being infinite, that Rule becomes $\frac{r\,f}{p\,r - f} = \varrho$, that is in Glaſs $\frac{r\,f}{2\,r - f} = \rho$, whence if f be greater than $2\,r$, ϱ becomes Negative, and $\frac{r\,f}{f - 2\,r}$ is the *Radius* of the Concave ſought.

Thoſe that are wholly to begin with this Dioptrical Science cannot do better than to read with Attention a late Treatiſe of Dioptricks, publiſhed by *W. Molineux*, Eſq; R. S. S. who has at large ſhewn the Nature of

Optick Glaſſes, and the Conſtruction and Uſe of Microſcopes and Teleſcopes; and though ſome nicely Critical have endeavoured to ſpy faults, and to traduce the Book, yet having long ſince examined it with care, I affirm, that if I can judge, it hath but two things that with any Colour may be call'd faults; the one an over-careful acknowledgment of every Trifle the Author had received from other; and the other, that he labours to make eaſie this curious Subject, ſo little underſtood by moſt, in a manner perhaps too familiar for the *Learned Critick*, and which demonſtrates that it was writ *cum animo docendi*, both which require but very little Friendſhip or good Nature in the Reader, to paſs for Vertues in an Author.

But to return to our firſt Theorem, which accounting for the thickneſs of the *Lens*, we will here again reſume, *viz.*

$$\frac{mpdr\rho - nd\rho t + npr\rho t}{ndr \pm md\varrho - mpr\varrho - m - ndt + nrt} = f.$$

And let it be required to find the *focus* where a whole Sphere will collect the Beams proceeding from an Object at the diſtance d; Here t is equal to $2r$ and $r = \varrho$. And after due Reduction the Theorem will ſtand thus, $\frac{mpdr - 2ndr \pm 2npr r}{2nd \pm 2nr - mpr} = f$, but if d be infinite, it is contracted to $\frac{mpr}{2n} - r = \frac{2n-m}{2m-2n}r = f$, wherefore a Sphere of Glaſs collects the Suns Beams at half the Semi-diameter of the Sphere without it, and a Sphere of Water at a whole Semi-diameter. But if the *ratio* of Refraction m to n be as 2 to 1, the *focus* falls on the oppoſite ſurface of the Sphere, but if it be of greater inequality it falls within.

Ano-

Another Example ſhall be when a Hemiſphere is expoſed to parallel Rays, that is d and e being infinite, and $t = r$, and after due Reduction the Theorem reſults $\frac{n\,n}{m\,m - m\,n}\,r = f$. That is, in Glaſs it is at $\frac{3}{2}\,r$, in Water at $\frac{2}{1}\,r$, but if the Hemiſphere were Diamant, it would collect the Beams at $\frac{4}{15}$ of the *Radius* beyond the Center.

Laſtly, As to the effect of turning the two ſides of a *Lens* towards an Object; it is evident, that if the thickneſs of the *Lens* be very ſmall, ſo as that you neglect it, or account $t = 0$, then in all caſes the *focus* of the ſame *Lens*, to whatſoever Beams, will be the ſame, without any difference upon the turning the *Lens*: But if you are ſo Curious as to conſider the thickneſs, (which is ſeldom worth accounting for) in the caſe of parallel Rays falling on a *Plano-Convex* of Glaſs, if the plain ſide be towards the Object, t does occaſion no difference, but the focal diſtance $f = 2\,r$. But when the Convex ſide is towards the Object, it is contracted to $2\,r - \frac{2}{3}\,t$, ſo that the *focus* is nearer by $\frac{2}{3}\,t$. If the *Lens* be double Convex the difference is leſs; if a *Meniſcus* greater. If the Convexity on both ſides be equal, the focal length is about $\frac{1}{6}\,t$ ſhorter than when $t = 0$. In a *Meniſcus* the Concave ſide towards the Object encreaſes the focal length, but the Convex towards the Object diminiſhes it. A General Rule for the difference ariſing on turning the *Lens*, where the *Focus* is Affirmative, is this $\frac{2\,r\,t - 2\,e\,t}{3\,r + 3\,e - t}$, for double Convexes of differing Spheres. But for *Menifci* the ſame difference becomes $\frac{2\,r\,t + 2\,e\,t}{3\,r - 3\,e + t}$; of which I need give no other demonſtration, but that by a due Reduction it will ſo follow from what is premiſed, as will the Theorems for all ſorts of Problems relating to the *foci* of Optick Glaſſes.